

INVERSE QUANTUM STATES OF HYDROGEN

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ABSTRACT

The possible existence of fractional quantum states in the hydrogen atom has been debated since the advent of quantum theory in 1924. Interest in the topic has intensified recently due to the claimed experimental findings of Randell Mills at Blacklight Power, Inc., Cranbury, New Jersey of 137 inverse principal quantum levels, which he terms the “hydrino” state of hydrogen. This paper will show that the classic wave equation predicts exactly that number of reciprocal energy states.

Keywords : inverse states, atomic refractive index, half-integral orbital angular momentum

INTRODUCTION:

Intensive laboratory research over much of the past decade at Blacklight Power and at the Technical University of Eindhoven, see Ref. [2] for a review, on what has come to be known as the “hydrino” state of hydrogen has sent theorists scurrying to explain the experimental spectroscopic observations on the basis of known and trusted physical laws. Jan Naudts of the University of Antwerp, for example, offers the argument that the Klein-Gordon equation of relativistic quantum mechanics allows an inverse quantum state.¹ Andreas Rathke of the European Space Agency’s Advanced Concepts Team insists that the classic wave equation cannot produce square integrable fractional quantum states.² Randell Mills of Blacklight Power employs a theory based on the Bohr concept of a particle in orbit around a much heavier central mass.³ Based on the results of the following analysis, Naudts and Mills appear to be on the right track.

PRESENTATION:

The classic matter wave equation

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad (1)$$

in spherical coordinates r, θ, ϕ , can be written as

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \left\{ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2} \right\} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad (2)$$

where v is the matter-wave phase velocity.

Calculation is eased by use of the transformation

$$\mu = \cos \theta \quad (3)$$

which produces a simplified equation

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \left\{ \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \psi}{\partial \mu} \right] + \frac{1}{1 - \mu^2} \frac{\partial^2 \psi}{\partial \phi^2} \right\} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad (4)$$

where r is the distance of the electron from the nucleus.

Suppression of the time in equation (4) can be achieved by use of the substitution

$$\psi = \psi_o e^{\pm i\omega t} \quad (5)$$

producing

$$\frac{\partial^2 \psi_o}{\partial r^2} + \frac{2}{r} \frac{\partial \psi_o}{\partial r} + \frac{1}{r^2} \left\{ \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \psi_o}{\partial \mu} \right] + \frac{1}{1 - \mu^2} \frac{\partial^2 \psi_o}{\partial \phi^2} \right\} + \frac{\omega^2}{v^2} \psi_o = 0 \quad (6)$$

Separation of the space variables can be achieved by the ansatz

$$\Psi = R\Theta\Phi \quad (7)$$

where each factor in the product is a function of the variable represented. When transformation (7) is used in equation (6), and the resulting equation is divided by the product given, produced is

$$\frac{1}{R} \frac{\partial^2 R}{\partial r^2} + \frac{2}{Rr} \frac{\partial R}{\partial r} + \frac{1}{r^2} \left\{ \frac{1}{\Theta} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \Theta}{\partial \mu} \right] + \frac{1}{\Phi(1 - \mu^2)} \frac{\partial^2 \Phi}{\partial \phi^2} \right\} + \frac{\omega^2}{v^2} = 0 \quad (8)$$

The term containing the equatorial angle ϕ can be separated and shown to be a constant

$$\frac{1}{\Phi} \frac{d^2 \Phi}{d\phi^2} = -m^2 \quad (9)$$

resulting in the differential equation

$$\frac{d^2 \Phi}{d\phi^2} + m^2 \Phi = 0 \quad (10)$$

Solutions of equation (10) are found by

$$\Phi = e^{\pm im\phi} \quad (11)$$

(except for a multiplying constant), where m is the orbital magnetic quantum number with values $0, \pm 1, \pm 2, \pm 3, \pm \dots$

The quantity within braces in equation (8) can now be written as

$$\frac{1}{\Theta} \frac{d}{d\mu} \left[(1 - \mu^2) \frac{d\Theta}{d\mu} \right] - \frac{1}{1 - \mu^2} m^2 = -k^2 \quad (12)$$

which, when both sides are multiplied by Θ , yields

$$\frac{d}{d\mu} \left[(1 - \mu^2) \frac{d\Theta}{d\mu} \right] - \frac{m^2 \Theta}{1 - \mu^2} + k^2 \Theta = 0 \quad (13)$$

Multiplying each term by $1 - \mu^2$,

$$(1 - \mu^2) \frac{d}{d\mu} \left[(1 - \mu^2) \frac{d\Theta}{d\mu} \right] - m^2 \Theta + (1 - \mu^2) k^2 \Theta = 0 \quad (14)$$

A solution in the form of the Maclaurin series

$$\Theta = a_l \mu^l \quad (15)$$

leads to

$$k^2 = l(l + 1) \quad (16)$$

because coefficients higher than a_l are zero.

The equation (8) containing the variable r can now be written

$$\frac{1}{R} \frac{d^2 R}{dr^2} + \frac{2}{Rr} \frac{dR}{dr} + \frac{1}{r^2} [-k^2] + \frac{\omega^2}{v^2} = 0 \quad (17)$$

When the value of k^2 in equation (16) is substituted in equation (17), obtained is

$$\frac{1}{R} \frac{d^2 R}{dr^2} + \frac{2}{Rr} \frac{dR}{dr} + \frac{1}{r^2} [-l(l+1)] + \frac{\omega^2}{v^2} = 0 \quad (18)$$

which provides

$$\frac{1}{R} \frac{d^2 R}{dr^2} + \frac{2}{Rr} \frac{dR}{dr} + \left[\frac{\omega^2}{v^2} - \frac{l(l+1)}{r^2} \right] = 0 \quad (19)$$

Multiplying each term by R yields

$$\frac{d^2 R}{dr^2} + \frac{2}{r} \frac{dR}{dr} + \left[\frac{\omega^2}{v^2} - \frac{l(l+1)}{r^2} \right] R = 0 \quad (20)$$

and, after extraction of r^2 from the bracket, leaves

$$\frac{d^2 R}{dr^2} + \frac{2}{r} \frac{dR}{dr} + \left[\frac{(r\omega)^2}{v^2} - l(l+1) \right] \frac{R}{r^2} = 0 \quad (21)$$

The electron spin is found from the series representation

$$R = a_s r^s \quad (22)$$

which produces an electron spin of $[-\frac{1}{2}]$. To obtain the other spin, we employ the series

$$R = a_s r^{-s} \quad (23)$$

which produces spin of $[+ \frac{1}{2}]$

To obtain inverse energy levels, which is to say below-groundstate energy levels, we suppress the spin in equation (21) by use of $R = ar^{-1/2}$, which results in

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \left[\frac{(r\omega)^2}{v^2} - (l + \frac{1}{2})^2 \right] \frac{R}{r^2} = 0 \quad (24)$$

For circular orbits in the subground region, the orbital angular momentum number l is $\frac{1}{2}$, which renders the total angular momentum $l + \frac{1}{2}$ equal to unity. Further, since the limiting value of the phase velocity is the speed of light c , equation (24) can now be rewritten as

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \left[\frac{v^2}{c^2} - 1 \right] \frac{R}{r^2} = 0 \quad (25)$$

The reciprocal quantum energy levels are found from the series representation

$$R = a_p r^{\frac{1}{p}} \quad (26)$$

After first and second derivatives are found and placed in equation (25), obtained is

$$\frac{1}{p^2} + \frac{v^2}{c^2} - 1 = 0 \quad (27)$$

which produces

$$\frac{1}{p} = \sqrt{1 - \frac{v^2}{c^2}} \quad (28)$$

where p is the index of refraction very near the nucleus. The principal quantum number n is the ratio $\frac{v}{c}$, with a range $\frac{1}{137} \leq n \leq 1$. For $n = \frac{1}{137}$, the electron velocity is 2.18×10^6 m/sec.

CONCLUSION: The standard wave equation indicates that fractional quantum states exist. The solution is square integrable, satisfying a fundamental tenet of quantum physics. Mills' claim of 137 different inverse energy levels seems confirmed, as is Naudts' relativistic analysis showing that at least one reciprocal state can exist.

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REFERENCES:

- [1] Naudts J., (2005) On the hydrino state of the relativistic hydrogen atom, physics/0507193 2
- [2] Rathke A., (2005) A critical analysis of the hydrino model *New Journal of Physics* 7 127
- [3] Mills R. The Grand Unified Theory of Classical Quantum Mechanics <http://www.blacklightpower.com/theory/book.shtml>